Domain Decomposition for Problems of Hemodynamics

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Motivation

Finite element simulations of arterial blood flow serve to provide pressure, velocity pattern and wall shear stress in regions of arterial dilatations and bifurcations. Even moderate size models require solving linear systems with millions of unknowns; 20 hours per time step is an average sequential run time, so parallel computing is a necessity. The main idea behind the actual research is to compare various spatial domain decomposition (4) strategies, perfectly suited for the MPI/OpenMP hybrid programming mode.

Method

We use non-overlapping domain decomposition as a basis for distributed computing. Local subproblems are solved repeatedly together with the continuity constraints across the interfaces, until global convergence is satisfied. Furthermore, appropriate level of spatial resolution could be specified for each subdomain. Figures 1 and 2 illustrate the idea. We work with non-matching discretizations, and we are interested to study the influence of

- different types of transmission conditions
- geometric discontinuities across contact regions
- extension operators to evaluate continuity jumps

Within each subdomain we solve the Navier-Stokes equations, using the method of characteristics (3) to approximate the non-linear convective term

$$\int_\Omega a\boldsymbol{u}\cdot\boldsymbol{v}+\nu\nabla(\boldsymbol{u}):\nabla(\boldsymbol{v})-p\,\nabla\cdot\boldsymbol{v}=\int_\Omega \boldsymbol{f}\cdot\boldsymbol{v}+$$

$$+\int_{\Gamma_N}\boldsymbol{g}_N\cdot\boldsymbol{v}+\int_{\Gamma_{ij}}\boldsymbol{T}(\boldsymbol{u},p)\boldsymbol{n}\cdot\boldsymbol{v},\quad\int_\Omega q\nabla\cdot\boldsymbol{u}=0$$

where $(\boldsymbol{u},\boldsymbol{p})$ stands for the velocity pressure couple, $(\boldsymbol{f},\boldsymbol{g}_N)$ - body force and boundary tractions, a,ν are positive constants, $(q,\boldsymbol{v})$ - admissible test functions. On the interface Γ_{ij} the continuity of the Cauchy stress $\boldsymbol{T}(\boldsymbol{u},p)$ is satisfied in weak form.

FreeFem++ on HECToR XT6

Open source http://www.freefem.org finite element code was installed on HECToR and Ness. The actual version supports MPI execution mode only. We coded OpenMP sparse matrix vector product, essential for iterative solvers; its performance is illustrated in Figure 3, speedup $\times 18.6$. The aneurysm model (2) was partitioned into 5 subdomains; each one being handled by 3 nodes. Scalar velocity fields are solved using 24 OpenMP threads. An additional MPI process recovers global continuity. This leads to

$$\textit{Total CPU cores} = 5\cdot 3\cdot 24 + 1\cdot 24 = 384$$

We implemented the Dirichlet-Neumann algorithm, Figure 2, that requires 120 steps to converge, $\varepsilon < 10^{-8}$ and we are actually focused on FETI (1) method. Even though FreeFem++ contains sequential sections, we observe significant parallel scaling.

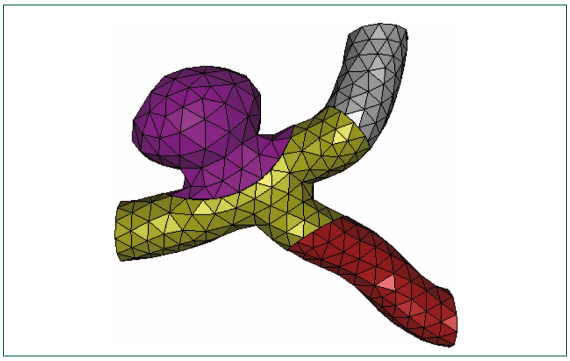

Fig. 1. – Non-overlapping domain decomposition of a saccular aneurysm model; very coarse discretization.

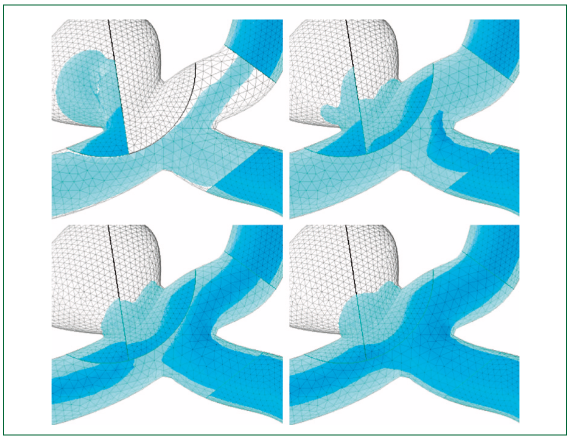

Fig. 2. – Computed velocity solution at iterations 2, 12, 29, 120; iso-surface 90 mm/s.

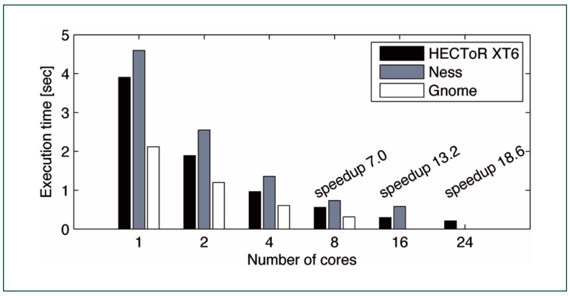

Fig. 3. – Execution time in seconds of a sparse matrix vector product implemented in FreeFem++ using OpenMP: *dim* (*A*) =2095119, max. 52 entries per row.

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